

Some sufficient conditions for p -nilpotence of a finite group

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ABSTRACT – In this paper, we give some new characterizations of finite p -nilpotent groups by using the notion of \mathcal{HC} -subgroups and extend several recent results.

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1. Introduction

In the present paper, we consider only finite groups. We use conventional notions and notation, as in Huppert (see [9]). G always denotes a finite group, $|G|$ is the order of G , p denotes a fixed prime, \mathfrak{U} is the class of all supersoluble groups and $Z_{\mathfrak{U}}(G)$ is the product of all the normal subgroups of G whose G -chief factors have prime order. A normal subgroup E of G is said to be hypercyclically (resp. p -hypercyclically) embedded in G if every chief factor (resp. p -chief factor) of G below E is cyclic. If G/L is a supersoluble (resp. p -supersoluble), then G is supersoluble (resp. p -supersoluble) if and only if L is hypercyclically (resp. p -hypercyclically) embedded in G .

A subgroup H of G is said to be C -normal in G if G has a normal subgroup T such that $G = HT$ and $H \cap T \leq H_G$, where H_G is the normal core of H in G (see [16]). A subgroup H of G is said to be an \mathcal{H} -subgroup of G if $H^g \cap N_G(H) \leq$

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H for all $g \in G$ (see [3]). Many people studied the structure of finite groups based on those two concepts and a lot of research has been given; see for example [1, 2, 3, 4, 6, 7, 11, 15, 16]. Recently, Wei and Guo (see [18]) introduced the following concept:

DEFINITION 1.1. A subgroup H of G is said to be an \mathcal{HC} -subgroup of G if there exists a normal subgroup T of G such that $G = HT$ and $H^g \cap N_T(H) \leq H$ for all $g \in G$.

It is clear that each of C -normal subgroup and \mathcal{H} -subgroup imply that \mathcal{HC} -subgroup. The converse does not hold in general, see Examples 1 and 2 in [18]. In [17, 18], some conditions for a group to be supersolvable are given and many known results are generalized. In this paper, we give some new criteria for p -nilpotence of a finite group by assuming that some kind of subgroups having some fixed prime power order are \mathcal{HC} -subgroups.

2. Preliminaries

LEMMA 2.1 ([18, Lemma 2.2]). *Suppose that H is an \mathcal{HC} -subgroup of G .*

(1) *If $H \leq K \leq G$, then H is an \mathcal{HC} -subgroup of K .*

(2) *If $N \trianglelefteq G$ and $N \leq H \leq G$, then H/N is an \mathcal{HC} -subgroup of G/N .*

(3) *If H is a p -subgroup and N is a normal p' -subgroup of G , then HN is an \mathcal{HC} -subgroup of G and HN/N is an \mathcal{HC} -subgroup of G/N .*

PROOF. (1) and (2) is [18, Lemma 2.3]. (3) is [18, Lemma 2.4]. □

LEMMA 2.2 ([17, Lemma 2.8]). *Let p be the smallest prime dividing $|G|$ and P a Sylow p -subgroup of G . If P is cyclic or P has a subgroup D with $1 < |D| < |P|$ such that every subgroup of P of order $|D|$ or 4 (if $|D| = 2$) is an \mathcal{HC} -subgroup of G , then G is p -nilpotent.*

LEMMA 2.3. *Let P be a nontrivial normal p -subgroup of G . If there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup of P of order $|D|$ or 4 (if $|D| = 2$) is C -normal in G , then $P \leq Z_{\mathcal{U}}(G)$.*

PROOF. It is a corollary of [13, Theorem]. □

LEMMA 2.4 ([18, Lemma 2.5]). *Let K be a normal subgroup of G and H a normal subgroup of K . If H is an \mathcal{HC} -subgroup of G , then H is C -normal in G .*

LEMMA 2.5 ([3, Theorem 6 (2)]). *Let H be an \mathcal{H} -subgroup of G . If H is subnormal in G , then H is normal in G .*

LEMMA 2.6. *Let P be a nontrivial normal p -subgroup of G . If there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup of P of order $|D|$ or 4 (if $|D| = 2$) is an \mathcal{H} -subgroup of G , then $P \leq Z_{\mathcal{U}}(G)$.*

PROOF. By Lemma 2.5, every subgroup of P of order $|D|$ or 4 (if $|D| = 2$) is normal in G . In view of Lemma 2.3, $P \leq Z_{\mathfrak{U}}(G)$. \square

LEMMA 2.7. *Let P be a nontrivial normal p -subgroup of G . If every maximal subgroup of P is an \mathcal{HC} -subgroup of G , then $P \leq Z_{\mathfrak{U}}(G)$.*

PROOF. By Lemma 2.4, every maximal subgroup of P is C -normal in G . In view of Lemma 2.3, $P \leq Z_{\mathfrak{U}}(G)$. \square

LEMMA 2.8. *Let P be a nontrivial normal p -subgroup of G . If there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup of P of order $|D|$ or 4 (if $|D| = 2$) is an \mathcal{HC} -subgroup of G , then $P \leq Z_{\mathfrak{U}}(G)$.*

PROOF. If every subgroup of P of order $|D|$ or 4 (if $|D| = 2$) is an \mathcal{H} -subgroup of G , then $P \leq Z_{\mathfrak{U}}(G)$ by Lemma 2.6. Hence we may assume that there exists a subgroup H of P with $|H| = |D|$ such that H is not an \mathcal{H} -subgroup of G . By hypothesis, there exists a proper normal subgroup K of G such that $G = HK$ and $H^g \cap N_K(H) \leq H$ for all $g \in G$. Then we can pick a normal subgroup M of G such that $K \leq M$ and $|G : M| = p$. Obviously, $P \cap M$ is a maximal subgroup of P . If $|P : D| = p$, then $P \leq Z_{\mathfrak{U}}(G)$ by Lemma 2.7. Hence we may assume that $|P : D| > p$. Then every subgroup of $P \cap M$ of order $|D|$ or 4 (if $|D| = 2$) is an \mathcal{HC} -subgroup of G . By induction, $P \cap M \leq Z_{\mathfrak{U}}(G)$. Since $|P/P \cap M| = p$, it follows that $P \leq Z_{\mathfrak{U}}(G)$. \square

LEMMA 2.9 ([18, Theorem 3.3]). *Let P be a Sylow p -subgroup of G . Then G is p -nilpotent if and only if $N_G(P)$ is p -nilpotent and every maximal subgroup of P is an \mathcal{HC} -subgroup of G .*

LEMMA 2.10 ([5, Theorem 8.3.1]). *Let P be a Sylow p -subgroup of G , where p is an odd prime divisor of $|G|$. Then G is p -nilpotent if and only if $N_G(Z(J(P)))$ is p -nilpotent, where $J(P)$ is the Thompson subgroup of P .*

LEMMA 2.11 ([8, Lemma 3.3]). *If G is p -supersoluble and $O_{p'}(G) = 1$, then G is supersoluble.*

For any group G , the generalized Fitting subgroup $F^*(G)$ is the set of all elements x of G which induce an inner automorphism on every chief factor of G . Clearly, $F^*(G)$ is a characteristic subgroup of G (see [10, X, 13]).

LEMMA 2.12 ([14, Theorem C]). *Let E be a normal subgroup of G . If $F^*(E)$ is hypercyclically embedded in G , then E is also hypercyclically embedded in G .*

In the following, we shall denote by $F_p(G)$ the p -Fitting subgroup of G . In fact, $F_p(G) = O_{p'}(G)$.

LEMMA 2.13. *A p -soluble normal subgroup E of G is p -hypercyclically embedded in G if and only if $F_p(E)$ is p -hypercyclically embedded in G .*

PROOF. We only need to prove the sufficiency. Suppose that the assertion is false and let (G, E) be a counterexample with $|G||E|$ minimal. We claim that $O_{p'}(E) = 1$. Indeed, since $F_p(E/O_{p'}(E)) = F_p(E)/O_{p'}(E)$, it is easy to verify that the hypothesis of the lemma holds for $(G/O_{p'}(E), E/O_{p'}(E))$. If $O_{p'}(E) \neq 1$, then the minimal choice of (G, E) implies that $E/O_{p'}(E)$ is p -hypercyclically embedded in $G/O_{p'}(E)$. Clearly $O_{p'}(E)$ is p -hypercyclically embedded in G . Therefore, E is p -hypercyclically embedded in G , a contradiction. Since E is p -soluble and $O_{p'}(E) = 1$, it follows that $F^*(E) = F(E) = F_p(E) = O_p(E)$, and so $F^*(E)$ is hypercyclically embedded in G . Applying Lemma 2.12, E is hypercyclically embedded in G , a contradiction again. \square

3. Main Results

THEOREM 3.1. *Let L be a p -soluble normal subgroup of G such that G/L is p -supersoluble, where p is a prime divisor of $|G|$. Suppose that for a Sylow p -subgroup P of $F_p(L)$, there exists a subgroup D of P such that $1 < |D| < |P|$ and every subgroup H of P with $|H| = |D|$ (and order 4 if $|D| = 2$) is an \mathcal{HC} -subgroup of G . Then G is p -supersoluble. In particular, if p is the smallest prime divisor of $|G|$, then G is p -nilpotent.*

PROOF. We distinguish two cases:

Case I: $O_{p'}(L) \neq 1$.

We consider the factor group $G/O_{p'}(L)$. Obviously, $(G/O_{p'}(L))/(L/O_{p'}(L)) \cong G/L$ is p -supersoluble. Since $O_{p'}(L/O_{p'}(L)) = 1$, we have

$$F_p(L/O_{p'}(L)) = O_p(L/O_{p'}(L)) = F_p(L)/O_{p'}(L) = PO_{p'}(L)/O_{p'}(L).$$

In view of Lemma 2.1(3), every subgroup of $F_p(L/O_{p'}(L))$ with order $|D|$ is an \mathcal{HC} -subgroup of $G/O_{p'}(L)$. Thus $G/O_{p'}(L)$ satisfies the hypothesis of the theorem. By induction, we have $G/O_{p'}(L)$ is p -supersoluble and so G is p -supersoluble.

Case II: $O_{p'}(L) = 1$.

Obviously, $F_p(L) = F(L) = O_p(L) = P$. Applying Lemma 2.8, $F_p(L)$ is hypercyclically embedded in G . In particular, $F_p(L)$ is p -hypercyclically embedded in G . In view of Lemma 2.13, L is p -hypercyclically embedded in G . Since G/L is p -supersoluble by hypothesis, it follows that G is p -supersoluble. \square

THEOREM 3.2. *Let L be a normal subgroup of G such that G/L is p -supersoluble, where p is the smallest prime divisor of $|L|$. Suppose that for a Sylow p -subgroup P of L , there exists a subgroup D of P such that $1 < |D| < |P|$ and every subgroup H of P with $|H| = |D|$ (and order 4 if $|D| = 2$) is an \mathcal{HC} -subgroup of G . Then G is p -supersoluble. In particular, if p is also the smallest prime divisor of $|G|$, then G is p -nilpotent.*

PROOF. By Lemma 2.1(1), it is easy to see that every subgroup H of P with $|H| = |D|$ (and order 4 if $|D| = 2$) is an \mathcal{HC} -subgroup of L . Applying Lemma 2.2, L is p -nilpotent. Then $O_{p'}(L)$ is the normal Hall p' -subgroup of L .

We distinguish two cases:

Case I: $O_{p'}(L) \neq 1$.

We consider the factor group $G/O_{p'}(L)$. Obviously, $(G/O_{p'}(L))/(L/O_{p'}(L)) \cong G/L$ is p -supersoluble. In view of Lemma 2.1(3), every subgroup of $PO_{p'}(L)/O_{p'}(L)$ with order $|D|$ is an \mathcal{HC} -subgroup of $G/O_{p'}(L)$. Thus $G/O_{p'}(L)$ satisfies the hypothesis of the theorem. By induction, we have $G/O_{p'}(L)$ is p -supersoluble and so G is p -supersoluble.

Case II: $O_{p'}(L) = 1$.

Then L is a normal p -subgroup of G . Applying Lemma 2.8, L is hypercyclically embedded in G . Since G/L is p -supersoluble by hypothesis, it follows that G is p -supersoluble. \square

THEOREM 3.3. *Let p be an odd prime divisor of $|G|$. Suppose that G has a normal subgroup L such that G/L is p -nilpotent and P is a Sylow p -subgroup of L . If there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with $|H| = |D|$ is an \mathcal{HC} -subgroup of G and $N_G(P)$ is p -nilpotent, then G is p -nilpotent.*

PROOF. Suppose that the theorem is false and let G be a counterexample of minimal order.

(1) $O_{p'}(G) = 1$.

Denote $T = O_p(G)$. If $T > 1$, consider G/T . It is obvious that $(G/T)/(LT/T) \cong G/LT$ is p -nilpotent. Let HT/T be a subgroup of PT/T with order $|D|$, where H is a subgroup of P with order $|D|$. Since H is an \mathcal{HC} -subgroup of G , HT/T is an \mathcal{HC} -subgroup of G/T by Lemma 2.1(3). Again, $N_{G/T}(PT/T) = N_G(P)T/T$ is p -nilpotent since $N_G(P)$ is p -nilpotent. Hence G/T satisfies the hypothesis of the theorem. The choice of G implies that G/T is p -nilpotent, and hence G is p -nilpotent, a contradiction.

(2) Let K be a proper subgroup of G such that with $P \leq K$. Then K is p -nilpotent.

By Lemma 2.1(1), every subgroup H of P with order $|D|$ is an \mathcal{HC} -subgroup of K . Since $N_K(P) \leq N_G(P)$ and $N_G(P)$ is p -nilpotent, it follows that $N_K(P)$ is p -nilpotent. Hence K satisfies the hypothesis of the theorem. Then K is p -nilpotent by the minimal choice of G .

(3) $L = G$.

If $L < G$, then L is p -nilpotent by step (2). Let T be the normal p -complement of L . Then $T \text{ char } L \trianglelefteq G$, so $T \trianglelefteq G$ and $T = 1$ by step (1). It follows that $L = P$ and $G = N_G(P)$ is p -nilpotent, a contradiction.

(4) $O_p(G) \neq 1$.

Consider the group $Z(J(P))$, where $J(P)$ is the Thompson subgroup of P . If $N_G(Z(J(P))) < G$, then $N_G(Z(J(P)))$ is p -nilpotent by step (2). Then G

is p -nilpotent by Lemma 2.10, a contradiction. Hence $N_G(Z(J(P))) = G$ and $1 < Z(J(P)) \leq O_p(G) < P$.

(5) $G/O_p(G)$ is p -nilpotent. In particular, $G/O_p(G)$ is p -supersoluble.

Let $\bar{G} = G/O_p(G)$, $\bar{P} = P/O_p(G)$, $\bar{K} = Z(J(\bar{P}))$ and $G_1/O_p(G) = N_{\bar{G}}(Z(J(\bar{P})))$. Since $O_p(\bar{G}) = 1$, we have $N_{\bar{G}}(Z(J(\bar{P}))) < \bar{G}$. Thus $G_1 < G$. By step (2), we have G_1 is p -nilpotent. Then $N_{\bar{G}}(Z(J(\bar{P})))$ is p -nilpotent. Thus \bar{G} is p -nilpotent by Lemma 2.10.

(6) $G = PQ$, where Q is a Sylow q -subgroup of G with $p > q$.

Step (5) shows that G is p -soluble. Then there exists a Sylow q -subgroup Q of G such that PQ is a subgroup of G for any $q \in \pi(G)$ with $q \neq p$ by [5, Theorem 6.3.5]. If $PQ < G$, then PQ is p -nilpotent by step (1). Hence $Q \leq C_G(O_p(G)) \leq O_p(G)$ by [12, Theorem 9.3.1], a contradiction. Thus $PQ = G$. By virtue of Lemma 2.2, $p > q$.

(7) $|P| > p|D|$.

This follows from Lemma 2.9.

(8) $O_p(G)$ is a maximal subgroup of P .

By step (5), we may assume that $G/O_p(G)$ has a normal Hall p' -subgroup $T/O_p(G)$. Obviously, T is normal in G and G/T is p -group. Then there exists a normal subgroup M of G such that $T \leq M$ and $|G : M| = p$. It is easy to see that $P \cap M$ is a maximal subgroup of P and also a Sylow p -subgroup of M . If $N_G(P \cap M) < G$, then, by step (1), $N_G(P \cap M)$ is p -nilpotent and so is $N_M(P \cap M)$. From step (7) and Lemma 2.1(1), every subgroup H of $P \cap M$ with order $|D|$ is an \mathcal{HC} -subgroup of M . Consequently, M satisfies the hypothesis of our theorem. Hence M is p -nilpotent by the minimal choice of G . Then G is p -nilpotent. This contradiction shows that $P \cap M$ is a normal p -subgroup of G . Since $O_p(G) < P$, it follows that $P \cap M = O_p(G)$ and so $O_p(G)$ is a maximal of P .

(9) $O_p(G)$ is hypercyclically embedded in G .

By steps (7) and (8), $|D| < |O_p(G)|$. By the hypothesis of the theorem, every subgroup H of $O_p(G)$ with order $|D|$ is an \mathcal{HC} -subgroup of G . Applying Lemma 2.8, we have step (9).

(9) G is supersoluble.

Since $G/O_p(G)$ is p -supersoluble and $O_p(G)$ is hypercyclically embedded in G , it follows that G is p -supersoluble. By Lemma 2.11 and step (1), G is supersoluble.

(10) Final contradiction.

Since G possesses a Sylow tower of supersolvable type, it follows that P is normal in G by step (6). Therefore, $G = N_G(P)$ is p -nilpotent by hypothesis, a contradiction. \square

THEOREM 3.4. *Let p be an odd prime divisor of $|G|$. Suppose that G has a normal subgroup L such that G/L is p -nilpotent and P is a Sylow p -subgroup of L . If there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with $|H| = |D|$ is an \mathcal{HC} -subgroup of G and $N_G(H)$ is p -nilpotent, then G is p -nilpotent.*

PROOF. We consider the following two case.

Case I: $L = G$.

Assume the theorem is not true and let G be a counterexample of minimal order. With a similar argument as in steps (1) and (2) of the proof of Theorem 3.3, we have the following steps (1) and (2).

(1) $O_{p'}(G) = 1$.

(2) Let K be a proper subgroup of G such that with $P \leq K$. Then K is p -nilpotent.

(3) P is a normal subgroup of G .

If $N_G(P) < G$, then $N_G(P)$ is p -nilpotent by step (2). Applying Theorem 3.3, G is p -nilpotent. This contradiction implies P is normal in G .

(4) P is hypercyclically embedded in G .

By hypothesis every subgroup H of P with order $|D|$ is an \mathcal{HC} -subgroup of G , then, from Lemma 2.8, (4) holds.

(5) Let N be a minimal normal subgroup of G . Then $|N| < |D| < |P|$.

In view of step (3), G is p -soluble. Then N is a p -subgroup by step (1) and so $N \leq P$. By virtue of step (4), $|N| = p$. If $|N| = |D|$, then $G = N_G(N)$ is p -nilpotent by the hypothesis of the theorem. This contradiction shows that $|N| < |D|$.

(6) Final contradiction.

By Lemma 2.1(2), it is easy to see that G/N satisfies the hypothesis of the theorem. Hence G/N is p -nilpotent by the minimal choice of G . Since the class of all p -nilpotent groups is a saturated formation, it follows that N is a unique minimal subgroup of G and $\Phi(G) = 1$. Consequently, $F(G) = N$. By steps (1) and (3), $F(G) = O_p(G) = P$. Hence $N = P$, contrary to step (5).

Case II: $L < G$.

By Lemma 2.1(1), every subgroup H of P with order $|D|$ is an \mathcal{HC} -subgroup of L . Obviously, $N_L(H)$ is p -nilpotent. By virtue of Case I, L is p -nilpotent. It follows that $L_{p'}$ is the normal Hall p' -subgroup of L . Clearly, $L_{p'} \trianglelefteq G$. If $L_{p'} \neq 1$, then it is easy to see that $G/L_{p'}$ satisfies the hypothesis of the theorem by virtue of Lemma 2.1(3). Hence $G/L_{p'}$ is p -nilpotent by induction. It follows that G is p -nilpotent. Hence we may assume that $L_{p'} = 1$. Then $L = P$. Since G/P is p -nilpotent, we may let V/P be the normal Hall p' -subgroup of G/P . By Schur-Zassenhaus Theorem, V has a Hall p' -subgroup $V_{p'}$. By Lemma 2.1(1), every subgroup H of P with order $|D|$ is an \mathcal{HC} -subgroup of V . Obviously, $N_V(H)$ is p -nilpotent. In view of Case I, $V = PV_{p'}$ is p -nilpotent and so $V_{p'}$ is normal in V . Obviously, $V_{p'}$ is also a normal p -complement of G and so G is p -nilpotent. \square

REMARK 3.5. Frobenius asserts that G is p -nilpotent if $N_G(H)$ is p -nilpotent for every p -subgroup H of G (see [9, Satz. IV.5.8]). In Theorem 3.4, we replace a condition of the Frobenius' theorem, namely, H is restricted to be a p -subgroup of a fixed order and we assume that H is an \mathcal{HC} -subgroup of G . Hence Theorem 3.4 can be considered as an extension of the Frobenius' theorem.

COROLLARY 3.6 ([1, Theorem 1.1]). *Let P be a Sylow p -subgroup of G . Then G is p -nilpotent if and only if $N_G(P)$ is p -nilpotent and every maximal subgroup*

of P is an \mathcal{H} -subgroup of G .

COROLLARY 3.7 ([6, Theorem 3.1]). *Let p be an odd prime dividing $|G|$ and P a Sylow p -subgroup of G . If every maximal subgroup of P is C -normal in G and $N_G(P)$ is p -nilpotent, then G is p -nilpotent.*

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